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Symmetry Breaking in a Generalized Skyrme Model

Joseph Schechter

Department of Physics, Syracuse University, Syracuse, NY

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SYMMETRY BREAKING IN A GENERALIZED SKYRME MODEL[1]

J. SCHECHTER

Department of Physics, Syracuse University,
Syracuse, NY 13244-1130,USA

ABSTRACT

We first outline the calculations of the neutron-proton mass difference and of the axial singlet matrix element (relevant to the "proton spin" puzzle) in a generalized Skyrme model of pseudoscalars and vectors. These two calculations are, perhaps surprisingly, linked to each other and furthermore are sensitive to some fine details of symmetry breaking in the effective meson Lagrangian. This provides a motivation for us to examine these symmetry breaking terms more closely. We find a qualitatively new feature in the symmetry breaking pattern of the vector meson system and discuss its significance.

1. Introduction

This material is based on work with Anand Subbaraman and Herbert Weigel which will be described in more detail elsewhere[1] The original motivation was to update some older papers[2,3,4], which should be consulted for adequate references to background work.

There are three reasons for taking the Skyrme model approach seriously. a) It *works* fairly well for baryon mass *differences*[5], static properties , scattering amplitudes etc. b) the picture is suggested by the consistent large N_c approximation scheme for QCD. c) It makes use of a nice feature of "old physics"– the "pion cloud" of the Yukawa theory – and builds upon it in a constructive way. Roughly speaking, the original Skyrme model replaces the "core" of the nucleon by a boundary condition. We shall consider here that the vector mesons describe more of the core physics. Other approaches utilize explicit quarks for this purpose.

The calculation of the non-electromagnetic part of the neutron-proton mass difference (as given in ref. 2) is reviewed in section 2. This has the interesting aspect that it forces one to go beyond the Skyrme model of pions to include at least the η

[1]Talk at Workshop on "Baryons as Skyrme Solitons", September 28-30, 1992; Siegen, Germany.

field and also some information (via the introduction of vector mesons, for example) about "short distance" physics in the model. The extension to the full SU(3) model of pseudoscalars and vectors is, in fact, indicated.

In section 3, the calculation [3,6] of the protons's axial singlet matrix element in the pseudoscalar-vector model is briefly reviewed and the connection with the so-called "proton-spin puzzle[7]" is given. It is shown how a conjectured decomposition of the matrix element into "matter" and "glue" parts is related[4] to the calculation of the $n-p$ mass difference in this model.

In order to improve the accuracy of the calculations discussed in sections 2 and 3, or at the least, to test their sensitivity to reasonable parameter changes it is necessary to employ the *full three flavor* pseudoscalar-vector model. The first step of carefully discussing symmetry breaking in the underlying meson Lagrangian is given in section 4. It is noted that the introduction of derivative type symmetry breaking terms for the vectors dramatically improves the agreement with experiment. This encourages us to suggest that the three-flavor chiral perturbation theory program[8] be extended to include the vectors as well as pseudoscalars so that it has a chance of describing low energy physics up to about 1 GeV. The relevant soliton calculations, using the full three flavor meson Lagrangian, are discussed in the following talk of Herbert Weigel[9].

2. Neutron-proton mass difference

We write

$$m(n) - m(p) = \Delta_{EM} + \Delta \tag{2.1}$$

where Δ_{EM} is due to photon exchange and Δ is due to the different u and d quark masses in the fundamental QCD Lagrangian. Subtracting the conventional estimate[10] for Δ_{EM} gives $\Delta = (2.05 \pm 0.30)$ MeV as the number to be understood in the present calculation. Let us first try to explain this at the two flavor level, which seems very reasonable. But there is a problem: In the original Skyrme model containing only pions, the mesonic term mocking up the fundamental iso-spin breaker has the form $\mathrm{Tr}[\tau_3(U+U^\dagger)]$ which vanishes identically when we plug in $U = \cos\omega + i\underline{n}\cdot\underline{\tau}\sin\omega$ as the general parameterization of a 2×2 unitary unimodular matrix. A way out at the two flavor level would appear to be to include also an isoscalar meson, $\eta \sim (\overline{u}u + \overline{d}d)/\sqrt{2}$. In this case U is no longer unimodular and the mesonic symmetry breaker does not vanish when we plug in the more general form $U = e^{i\chi}(\cos\omega + i\underline{n}\cdot\underline{\tau}\sin\omega)$. In particle language we get a symmetry breaker like $\epsilon\eta\pi^0$. We would expect an $n-p$ splitting in the Yukawa theory from graphs in which the nucleon emits a virtual π^0 which converts, due to the $\epsilon\eta\pi^0$ interaction, into an η which is then reabsorbed by the nucleon. Unfortunately the nucleon matrix elements of the operator $\epsilon\eta\pi^0$ vanish for the original Skyrme Lagrangian; the η field does not get excited. This is because the η field only appears quadratically in the Lagrangian and, for example, the static Hamiltonian terms $\frac{1}{2}(\nabla\eta)^2 + \frac{1}{2}m_\eta^2\eta^2$ are minimized for $\eta = 0$. Clearly a term linear in η is required to act as a source (the term $\epsilon\eta\pi^0$ does so to a small extent but leads to $\Delta = \mathcal{O}(\epsilon^2)$ which is negligible). It is interesting that exciting an η forces us to extend our original model. We may think of this effect as arising from the "core" of the nucleon, described, for example, by vector mesons. It was shown that in a model of pseudoscalars *and* vectors there are terms[11] linear in η proportional to the Levi-Civita tensor,

$$\epsilon_{\mu\nu\alpha\beta}\partial_\mu\eta(\omega_\nu\partial_\alpha\omega_\beta + \text{ many others}) \ .$$

These terms do excite the η field to the required strong order when account is taken of the need to "crank" the η field. This amounts to allowing the η to get excited by centrifugal effects; the ansatz

$$\eta(\underline{r}) = \eta(r)\hat{\underline{r}} \cdot \underline{\Omega}, \tag{2.2}$$

where $\frac{i}{2}\tau_a\Omega_a = A^\dagger\dot{A}$ is the angular velocity, (A is defined from $U(t) = A(t)U_c(\underline{x})A^\dagger(t)$, $U_c(x)$ being the classical soliton solution) is substituted in and the profile $\eta(r)$ is chosen so as to maximize the moment of inertia. Using (2.2) in the symmetry breaking terms finally gives an iso-spin breaking piece in the collective Hamiltonian of the form

$$H_{SB} = -\Delta I_3, \tag{2.3}$$

where I_3 is the iso-spin *operator*. Note that there will now be a number of different types of symmetry breaking terms in our effective meson Lagrangian. For example there will also be an $\omega\rho^0$ transition piece and others involving more derivatives. Several years ago, Δ was estimated[2] on this basis to be about 1.3 MeV, just a little low compared to (2.1). It was also shown that the extra amount was likely due to inclusion of the strange degree of freedom. This is plausible in the Yukawa theory since K meson exchange with unequal K^+ and K^0 meson masses is also expected to contribute to Δ.

However there are a number of interesting complications when we consider the three flavor case in the generalized Skyrme model approach. First, of course, we have both η and η' fields appearing. Structurally, the η' appears in the mesonic Lagrangian as the phase of the 3×3 matrix U in analogy to the η in the two flavor case. But there is now a qualitatively new feature in that one has a contribution to Δ even without cranking the η. This is proportioned to

$$< n|D_{38}(A)|n > - < p|D_{38}(A)|p >, \tag{2.4}$$

where $D_{ab}(A) = \frac{1}{2}$ Tr $(\lambda_a A \lambda_b A^\dagger)$ is the SU(3) adjoint representation matrix element. The λ_a are the Gell-Mann matrices. Evaluating (2.4) using for $|n>$ and $|p>$ the eigenstates of the symmetric (no symmetry breaking terms) collective Hamiltonian of the pseudoscalar only Skyrme model actually gives the fairly large value of about 1.3 MeV as the contribution to Δ. This might suggest that perhaps there was no need to excite the η field by extending the model to include vectors. However that is wrong. Yabu and Ando[12] showed that the baryon eigenstates are very sensitive to symmetry breaking. When the collective Hamiltonian is diagonalized *exactly* the contribution to Δ is reduced by a factor of $\frac{1}{2}$. Furthermore, improving the Yabu-Ando calculation by cranking the K mesons leads[13] to another factor of $\frac{1}{2}$ reduction (in the mod el of pseudoscalars only). Altogether we thus *estimate*

$$\Delta \approx 1.3 + (1.3)/4 \approx 1.6 \text{ MeV} \ , \tag{2.5}$$

where the first piece is due to cranking the η at the two flavor level and the second piece is due to (2.4). A similar estimate was presented in ref. 2. We stress that this is just an estimate since the first part was calculated in a two flavor model of vectors and pseudoscalars while the second was gotten from a three flavor model of pseudoscalars only. It is thus desirable to see what the result would be in a treatment using the full three flavor model with both pseudoscalars and vectors throughout. This is part of our motivation for a more through analysis of symmetry breaking in the full model.

3. Axial Singlet Matrix Element

The current interest in this "hot topic" will turn out to perhaps justify the complicated meanderings to which we have exposed the reader in the last section. The form factors of the ath flavor axial current in the proton are defined by

$$\sqrt{\frac{p_0 p_0' V^2}{M_N^2}} < p'|i\overline{q}_a \gamma_\mu \gamma_5 q_a|p >= i\overline{u}(\underline{p}')[\gamma_\mu \gamma_5 H_a(q^2) + \frac{iq_\mu}{2M_N}\gamma_5 \widetilde{H}_a(q^2)]u(\underline{p}). \tag{3.1}$$

The *singlet* form factors are defined by

$$H(q^2) = \sum_{a=1}^{3} H_a(q^2); \ \widetilde{H}(q^2) = \sum_{a=1}^{3} \widetilde{H}_a(q^2), \tag{3.2}$$

where $(1,2,3) = (u,d,s)$.

A great deal of excitement was caused by the EMC (European Muon Collaboration) experiment[14] on deep inelastic polarized μ - polarized p scattering. The result can be interpreted as a measurement of

$$\frac{4}{9}H_1(0) + \frac{1}{9}H_2(0) + \frac{1}{9}H_3(0). \tag{3.3}$$

Another linear combination of these form factors is known from neutron beta decay:

$$H_1(0) - H_2(0) \simeq 1.25, \tag{3.4}$$

while a third can be estimated from hyperon beta decay plus SU(3) flavor covariance:

$$R \equiv H_1(0) + H_2(0) - 2H_3(0) \approx 0.68. \tag{3.5}$$

For the first time it was possible to get information on the matrix element of the axial *singlet* current, J_μ^5. This is especially interesting since J_μ^5 obeys the *anomalous* divergence equation

$$\partial_\mu J_\mu^5 = 2i \sum_{a=1}^{3} m_a \overline{q}_a \gamma_5 q_a + \partial_\mu K_\mu, \tag{3.6}$$

where K_μ is the Chern-Simons current of the Yang-Mills theory. Note that $\partial_\mu J^5_\mu$ does not vanish when the m_a go to zero. From the EMC result for (3.3) together with (3.4) and (3.5) one gets:

$$H(0) = 0.03 \pm 0.18, \tag{3.7a}$$

$$H_3(0) = -0.22 \pm 0.06. \tag{3.7b}$$

Eq. (3.7a) is surprising since it is expected to be 1.0 in the naive non-relativistic quark model wherein $\frac{1}{2}H(0)$ is identified as the proton expectation value of the *quark spin* part of the total angular momentum operator. Also, (3.7b) is surprising since the "strangeness content" of the proton is expected to be small. It seems to us that, of these two surprising results, only the first is a reliable deduction from the measurement of (3.3). That is because the use of flavor SU(3) to obtain R in (3.5) is debatable. The SU(3) Skyrme model, in fact, suggests[15] much smaller values of R (while not giving big SU(3) deviations for the flavor *changing* currents which enter into the Cabibbo theory). Taking, somewhat arbitrarily, $R = 0.3$ would lead to the deduction from experiment

$$H(0) = 0.12 \pm 0.18, \tag{3.8a}$$

$$H_3(0) = -0.06 \pm 0.06. \tag{3.8b}$$

Eq. (3.8a) is *still* small and surprising while (3.8b) is no longer surprising!

Now in the Skyrme model containing just pseudoscalar fields, there is a nice result[16]:

$$H(0) = 0. \tag{3.9}$$

So, what is *mysterious* in the quark model is the *expected* feature in the Skyrme model. Let us try to understand this from a different[17] point of view. In the Skyrme model of psudoscalars the η' field enters as $U = \exp\left(\frac{2i\eta'}{\sqrt{3}F_\pi}\right)\widetilde{U}$, where $\det\widetilde{U} = 1$. By Noether's theorem the axial singlet current is simply

$$J^5_\mu = -\sqrt{3}F_\pi \frac{\partial\mathcal{L}}{\partial(\partial_\mu\eta')} = \sqrt{3}F_\pi\partial_\mu\eta', \tag{3.10}$$

where the second step holds because η' enters only quadratically in $\mathcal{L}$ (we just pick up its kinetic term for (3.10)). Now the result (3.9) is evident since J^5_μ is a pure gradient and hence proportional to q_μ in momentum space. From the form factor decomposition in (3.1) we see that (3.10) only contributes to the "induced" form factor $\widetilde{H}(q^2)$ and hence $H(q^2)$ must vanish in this simple model.

But this is troublesome from the point of view of the $n-p$ calculation described in the last section. There we *needed* a term linear in η' in order to excite the pseudoscalar iso-singlet. We saw that there *was* such a term when vectors were added; it can be written as

$$\mathcal{L} = ... - \frac{1}{\sqrt{3}F_\pi}\partial_\mu\eta'\widetilde{J}^5_\mu. \tag{3.11}$$

Here $\widetilde{J}^5_\mu$ is proportional to $\epsilon_{\mu\nu\alpha\beta}$ and is due to the vector fields. It can be identified, using (3.10), as a new (short distance) contribution to J^5_μ:

$$J^5_\mu = \sqrt{3} F_\pi \partial_\mu \eta' + \widetilde{J}^5_\mu. \tag{3.12}$$

Since $\widetilde{J}^5_\mu$ is not a pure gradient[18] it does contribute to $H(0)$. Does this destroy the nice prediction of the Skyrme model? A couple of years ago it was found that $H(0) \approx 0.30$ in this model.[3,6] This is still qualitatively small (and in agreement with experiment if (3.8a) is taken rather than (3.7a)).

Another aspect of this problem which the Skyrme model approach might illuminate concerns the so-called "2-component decomposition".[19] This is an attempt to make the small value of $H(0)$ intuitively plausible from the QCD parton model point of view. It is certainly legal to write:

$$< p|J^5_\mu|p > = < p|(J^5_\mu - K_\mu)|p > + < p|K_\mu|p > . \tag{3.13}$$

Since the first term on the right hand side is conserved when the light quark masses are neglected it is tempting to consider it, in some sense, the usual "matter" contribution and to be around the naive quark model value of 1. It is hoped that this would be largely cancelled by the second "glue" term. There has been considerable discussion of this possibility in the literature.[20] One criticism is that the decomposition (3.13) is not gauge invariant. It has been suggested that a gauge invariant and hence better way is to give an analogous decomposition based on a generalized Goldberger-Trieman relation. This reads[21]:

$$H(0) = \frac{\sqrt{3} F_\pi}{2 M_N} \left(g_{\eta' NN} - \sqrt{3} F_\pi m^2_{\eta'} g_{GNN} \right), \tag{3.14}$$

where $g_{\eta' NN}$ is the Yukawa coupling constant of the (pure SU(3) singlet) η' field with the nucleons while g_{GNN} is the Yukawa coupling constant for the composite glue field $G \equiv \partial_\mu K_\mu$. The first term is supposed to be the "matter" contribution, the second the "glue" piece. Now it is very difficult[22] to obtain a reliable value for $g_{\eta' NN}$ from experiment and even harder to get an experimental handle on g_{GNN}. However it is possible to make a theoretical estimate in the generalized Skyrme model of pseudovectors and vectors. We showed[4] that the existence of the second term in (3.14) corresponds to the presence of a new term,

$$\frac{t}{3 F^2_\pi m^2_{\eta'}} \partial_\mu G \widetilde{J}^5_\mu, \tag{3.15}$$

in the effective Lagrangian. t is a new dimensionless constant. The effect of this term is to provide an extra contribution to the η' excitation (the "auxiliary" field G must be eliminated by its equation of motion in terms of η' to see this) but *not* to the current, J^5_μ. This means that the η contribution to Δ in (2.3) should be multiplied by a factor of $(1-t)$. Thus we can try to improve the predicted value of Δ by adjusting t. This predicts the coupling constants in the two component decomposition as[4]:

$$g_{\eta' NN} = (1-t)\frac{2M_N H(0)}{\sqrt{3}F_\pi},$$

$$g_{GNN} = \frac{t}{t-1}\frac{g_{\eta' NN}}{\sqrt{3}F_\pi m_{\eta'}}. \tag{3.16}$$

Here $H(0)$ should still be taken to be 0.30 as discussed after (3.12). Finally, "improving" the prediction for Δ (assuming the central value, 2.05 MeV) gives[4] for (3.14):

$$\begin{aligned} H(0) = &\ \text{“matter''} + \text{“glue''} \\ 0.3 = &\ 0.4 - 0.1. \end{aligned} \tag{3.17}$$

This indicates a rather small gluonic as well as a small matter contribution. We should stress, that since this estimate is related to perhaps arcane details of symmetry breaking and η' excitation, (see the discussion at the end of section 2) it is desirable to investigate further the sensitivity of the result to these factors.

4. Back to the Meson Lagrangian

The first step in a more detailed investigation of symmetry breaking in the generalized Skyrme model is clearly to examine the underlying meson Lagrangian itself. Since the soliton energy is in the neighborhood of 1 GeV it would seem prudent for the meson Lagrangian to adequately describe mesonic physics up to this energy. This would imply that we include in addition to the pseudoscalars, the vectors (because, among other things, they are "there"). We will also include the composite glue field G in order to to simplify the implementation of the $U(1)$ axial anomaly. (A composite scalar glue field, $H = \Theta_{\mu\mu}$ can also, if desired, be introduced in order to conveniently implement the trace anomaly).

The dynamical variables we use in the Lagrangian are, in addition to the chiral field U, $\xi \equiv U^{1/2}$ and a vector meson nonet, ρ_μ related to linearly transforming objects A_μ^L and A_μ^R by

$$A_\mu^L = \xi\rho_\mu\xi^\dagger + \frac{i}{\tilde{g}}\xi\partial_\mu\xi^\dagger;\ A_\mu^R = \xi^\dagger\rho_\mu\xi + \frac{i}{\tilde{g}}\xi^\dagger\partial_\mu\xi. \tag{4.1}$$

The chiral symmetric and OZI rule conserving terms of the Lagrangian have been given in many places.[23] Here we shall concentrate our discussion on the symmetry breaking terms. For this purpose we want to introduce some notation associated with the mass terms of the fundamental QCD Lagrangian:

$$\mathcal{L}_{\text{mass}} = -m_u\overline{u}u - m_d\overline{d}d - m_s\overline{s}s \equiv -\hat{m}\overline{q}\mathcal{M}q, \tag{4.2}$$

where $\hat{m} = (m_u + m_d)/2$ and the dimensionless matrix $\mathcal{M}$ is written as

$$\mathcal{M} = y\lambda_3 + T + xS,$$

$$\lambda_3 = \text{ diag } (1, -1, 0), \; T = \text{ diag } (1, 1, 0), \; S = \text{ diag } (0, 0, 1),$$

$$x = m_s/\hat{m}, \; y = \frac{m_u - m_d}{m_u + m_d}. \tag{4.3}$$

In addition we note the redundant, but convenient, quantity[10],

$$R = \frac{m_s - \hat{m}}{m_d - m_u} = \frac{\text{“strange” splitting}}{\text{iso} - \text{spin splitting}}. \tag{4.4}$$

The interest of R lies in the fact that it may, in principle, be straightforwardly extracted from SU(3) multiplets other than 0^- and 1^-. We have introduced so much notation for $\mathcal{M}$ since all symmetry breaking terms will be taken to be proportional to it. Rather than writing down all symmetry breaking terms as in the CPT (chiral perturbation theory) program[8] of pseudoscalars only, we shall only write the ones we believe are most important. These are the ones which (except for those needed to solve the η' problem) obey the OZI rule. Incidentally, we may note that the OZI violating symmetry breaking terms turn out to have small coefficients in the CPT approach too. We implement the OZI rule in Okubo's original form[24]: all mesons are described by nonets and only terms which are a single trace in flavor space are included. This eliminates the “hairpin” diagrams. Without further ado we list the OZI rule conserving, symmetry breaking terms to be included in the effective Lagrangian:

$$\alpha' \text{ Tr } [\mathcal{M}(A^L_\mu U A^R_\mu + A^R_\mu U^\dagger A^L_\mu)] + \beta' \text{ Tr } [\mathcal{M}(\partial_\mu U \partial_\mu U^\dagger U + U^\dagger \partial_\mu U \partial_\mu U^\dagger)]$$
$$+\gamma' \text{ Tr } [\mathcal{M}(F^L_{\mu\nu} U F^R_{\mu\nu} + F^R_{\mu\nu} U^\dagger F^L_{\mu\nu})] + \delta' \text{ Tr } [\mathcal{M}(U + U^\dagger - 2)]$$
$$+\lambda'^2 \text{ Tr } [\mathcal{M}U^\dagger \mathcal{M}U^\dagger + \mathcal{M}U\mathcal{M}U - 2\mathcal{M}^2], \tag{4.5}$$

where

$$F^{L,R}_{\mu\nu} = \partial_\mu A^{L,R}_\nu - \partial_\nu A^{L,R}_\mu - i\tilde{g}[A^{L,R}_\mu, A^{L,R}_\nu].$$

The explanation for these terms is as follows. First, the δ' term is the usual, main symmetry breaker for the pseudoscalars. The α' term plays the same role for the vectors. However, noting (4.1), the α' term also supplies an undesirably large amount of derivative-type symmetry breaking for the pseudoscalars. Most of this is cancelled by the β' term. Together, the α' and β' terms enable us to fit both F_K and F_π. The γ' term is a derivative type symmetry breaker for the vectors. It was set to zero for simplicity in ref. 2 but we shall see here that it may be rather important. Finally, the $(\lambda')^2$ term, though of second order in $\mathcal{M}$, is argued to be of the same order as the β' term in the CPT program[8] ($\mathcal{M}$ counts as two derivatives). This is reasonable because the symmetry breaking for the pseudoscalars is relatively large. Such a term may be less important for the vectors but, in any event, it would have the form μ' Tr $(\mathcal{M}A^L_\mu \mathcal{M}A^R_\mu)$.

Now our task is to fit the quantities

$$\alpha', \beta', \gamma', \delta', \lambda'^2;\ \tilde{g}, m_v;\ x = \frac{m_s}{\hat{m}},\ y = \frac{m_u - m_d}{m_u + m_d},$$

where $\tilde{g}$ is the "gauge coupling constant" in (4.1) and m_v is a "bare" vector meson mass, from the experimental mass spectrum of 0^- and 1^- mesons, the 0^- decay constants and the $V \to \phi\phi$ decay widths. Since there are many unknowns one might think that the work could be simplified if we had reliable information on the quark mass ratios x and y from outside the 0^- and 1^- multiplets. Let us make a small *detour* to see if this is possible.

Certainly the best source of information is the baryon octet. One may make a perturbation around zero quark masses, $m_u = m_d = m_s = 0$ (chiral perturbation theory) or around, say, the symmetric point

$$m_u = m_d = m_s = \frac{1}{3}(m_u + m_d + m_s), \tag{4.6}$$

which we may denote as "flavor" perturbation theory. We can then use group theory (generalized Wigner-Eckart theorem) to derive mass relations in a well known way. To first order in perturbation it doesn't matter which scheme is used and we find the Gell-Mann Okubo formula

$$\begin{aligned} \Lambda - N &= \frac{1}{3}[2(\Xi - N) - (\Sigma - N)] \\ 176.9 \text{ MeV } &= 168.1 \text{ MeV }, \end{aligned} \tag{4.7}$$

(particle symbols stand for their masses) and also

$$\begin{aligned} R = \frac{m_s - \hat{m}}{m_d - m_u} &= \frac{\Xi - \Sigma}{n - p} = \frac{\Sigma - N}{\Xi^- - \Xi^0} \\ 60.7 \pm 9.1 &= 46.2 \pm 5.9. \end{aligned} \tag{4.8}$$

In (4.8) the photon exchange contributions have been subtracted out of the iso-spin violating mass differences. The large deviation between the central values in (4.8) shows that the values of R indicated there may not be reliable. Together with the observed deviation in (4.7) this suggests that we go on to second order. There is a known difficulty[8,10] in going beyond first order in the CPT scheme since non-analytic terms like $m^{\frac{3}{2}}$ and $m \ln m$ arise. Since these are due to massless Goldstone boson exchange it would seem that this effect is already taken into account at zeroth order if we expand around the point (4.6). Recently, a new relation at second order was found[25]:

$$R = \frac{3\Lambda + \Sigma - 2N - 2\Xi}{2\sqrt{3}m_T + (n - p) + (\Xi^0 - \Xi^-)}, \tag{4.9}$$

where m_T is the "corrected" value of the $\Lambda - \Sigma^0$ transition mass. In principle, m_T may be determined from precision measurements of the difference between, say, $pK^- \to \Lambda\eta$

and $n\overline{K}^0 \to \Lambda\eta$. Thus (4.9) may be predictive in the future. For the moment the best we can do is to use the structure of (4.9) and the assumption that second order quantities don't deviate by more than 20% from their first order values to derive a lower bound,

$$R > 38 \pm 10. \tag{4.10}$$

However this isn't very precise and we are forced to end our detour by concluding that the quark mass ratios are not known well enough from "outside" to be useful for our purpose. The fundamental quark mass ratios should, in fact, thus be obtained as results from our fit.

The formulas for the meson one, two and three point functions used for fitting are fairly standard and are explicitly given in ref. 1. It is useful to keep $x = m_s/\hat{m}$ as a parameter during the fit and to evaluate everything for each value of x. Then once the other decay widths and masses are used as inputs we have four remaining predictions[1] for each x. These are displayed below with the experimental values indicated in parentheses:

x	$(K^{*0} - K^{*+})_{\mathrm{NON-EM}}$	$\frac{\Gamma(\rho\to 2\pi)}{\Gamma(K^*\to K\pi)}$	$\frac{\Gamma(\rho\to 2\pi)}{\Gamma(\phi\to K\overline{K})}$	$m(\phi)$
20	2.44 MeV	4.97	163	1.02 GeV (1.02)
25	3.05	4.45	81	1.04
30	3.97	3.90	57	1.09
32	4.47	3.67	47	1.13
36	5.84 $\begin{pmatrix} 5.2 \\ 7.4 \end{pmatrix}$	3.21 (3.0)	28 (40.3)	1.26

(Note that the experimental value for the non-electromagnetic part of the $K^{*0} - K^{*+}$ mass difference is 5.2 MeV if the individual masses given in the Particle Data Tables are simply subtracted but is 7.4 MeV if attention is restricted to the "dedicated" experiments.) We see that three quantities are well fit for x in the range $34 - 38$ but that $m(\phi)$ is fit for $x = 20$. A compromise "best fit" is characterized by the fundamental quark mass ratios:

$$x = 31.5, \qquad y = -0.42, \qquad R = 36. \tag{4.11}$$

This may be compared with the values obtained by Gasser and Leutwyler in their big Physics Report[10]:

$$x = 25.0 \pm 2.5, \qquad y = -0.28 \pm 0.03, \qquad R = 43.5 \pm 2.2. \tag{4.12}$$

As a kind of check we note that we would also get y= -0.27 when we set $x = 25$. Note that a recent determination[26] using ψ decays finds $y = -0.54 \pm 0.09$ which is much closer to our result than to (4.12).

To see what physics lies behind this "numerology" let us attempt to understand why our result (4.11) differs from (4.12). We remark that the Gasser-Leutwyler values are essentially what one gets by applying first order perturbation theory to the vector multiplet. However, there are some problems with this. For one thing the SU(3) relation between $(M_{\rho\omega})_{\rm NON-EM}$, the quark mass induced part of the $\rho\omega$ transition mass, and $(K^{*0} - K^{*+})_{\rm non-EM}$ appears to be very badly broken. In fact, they used[10] $(M_{\rho\omega})_{\rm non-EM}$ as an input and assumed something was wrong with the extraction of $(K^{*0} - K^{*+})_{\rm non-EM}$ from experiment. Furthermore even though the width relation between $\Gamma(K^* \to K\pi)$ and $\Gamma(\rho \to \pi\pi)$ was one of the historical successes of SU(3) invariance, the measured width of the ρ has increased from 100 MeV to 150 MeV since those days and the simple SU(3) relation no longer holds. Also the ordinary SU(3) width relation between $\Gamma(\phi \to K\overline{K})$ and $\Gamma(\rho \to 2\pi)$ does not hold. In the present approach $(K^{*0} - K^{*+})$, $\Gamma(K^* \to K\pi)/\Gamma(\rho \to 2\pi)$ and $\Gamma(\phi \to K\overline{K})/\Gamma(\rho \to 2\pi)$ can be explained with the help of a relatively important derivative symmetry breaker for the vectors (the γ' term in (4.5)). This modifies the SU(3) relations via non-trivial wave-function renormalizations for vector mesons containing strange quarks; explicitly[1],

$$Z_\rho = 0.99, \qquad Z_{K^*} = 0.84, \qquad Z_\phi = 0.65 \tag{4.13}$$

for the best fit parameters (Note, the physical ρ field, ρ_p, is given by $\rho_p = Z_\rho \rho$ etc.). To fine tune our prediction for $m(\phi)$, more exotic terms and loop corrections should be included. In any case it is clear that the above feature represents a new qualitative effect for describing the vector nonet system.

For our purposes the η' -OZI rule violating-interactions are also very important. To mock up the axial anomaly and produce an η' mass we add the terms:

$$\frac{1}{\kappa}G^2 + \frac{i}{12}G \ \ln(\det U/\det U^\dagger) + n \ {\rm Tr}\ (\alpha_\mu)\ {\rm Tr}\ (\alpha_\mu) + i\epsilon' G \ {\rm Tr}\ [\mathcal{M}(U - U^\dagger)], \tag{4.14}$$

where $\alpha_\mu \equiv \partial_\mu U U^\dagger$ and $G = \partial_\mu K_\mu$. No kinetic energy terms are included for G so it acts as an auxiliary field and gets eliminated by its equation of motion:

$$G = \frac{\sqrt{3}\kappa}{8} F_\pi \eta' - \frac{i\kappa\epsilon'}{2}\ {\rm Tr}\ [\mathcal{M}(U - U^\dagger)], \tag{4.15}$$

which is to be substituted back into (4.14). The Lagrangian which results[27] when one keeps just the first two terms in (4.14) (i.e. setting $n = \epsilon' = 0$) has been known for quite a while to be able to explain the η' mass and the $\eta - \eta'$ mixing angle but not [28] the η mass. With the two additional terms[29] we are now able to also explain the η mass as well as $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$. Note that substituting (4.15) into (4.14) yields, among other things, a symmetry breaker of "type 7" but not one of "type 6" in the second order CPT program[8]. Thus the assumption that the OZI rule violating interactions are dominated by the psudoscalar iso-singlet channel (and given by (4.14)) evidently selects a unique "frame" from the one parameter family allowed by the Kaplan-Manohar ambiguity[30].

With the present Lagrangian explaining more features of low energy meson physics, we may feel more confident about investigating symmetry breaking in the soliton sector. This aspect is discussed in refs. 1 and 9. Qualitatively, the effect of the important γ' term in (4.5) is to decrease the relative contribution of the η excitation ($\propto \delta'$) to the $n-p$ mass difference while leaving the net prediction about the same. This has the consequence[1] that, while the "glue" contribution to $H(0)$ remains relatively small, it could be somewhat larger than the value in (3.17).

The question of symmetry breaking in the pseudoscalar-vector system is certainly of interest outside the soliton sector too. It seems interesting to also calculate loop diagrams and include other "second-order" interaction terms (although we believe we have the dominant symmetry breakers) to construct an analog of the CPT scheme which would be useful up to about 1 GeV. Of course, this covers a lot of physics so we expect that progress in this direction will be incremental or evolutionary in character.

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